# Tests of General Relativity: memory effects

Marc Besancon [1], Adrien Cogez [1] and Antoine Petiteau [1]

[1] *CEA Paris-Saclay University, IRFU/DPhP, Bat.141, 91191 Gif sur Yvette, France*

E-mail: marc.besancon@cea.fr, adrien.cogez@cea.fr, antoine.petiteau@cea.fr

Thanks to the timing of arrays of pulsars (PTA) with multiple radio telescopes over many years, we can now observe the Universe with gravitational wave in the nanohertz band. EPTA (Europe), NanoGRAV (US) and PPTA (Australia) now grouped with MeerKAT in IPTA, have shown strong evidence for a signal in this band, which SKAO will be able to characterise in detail. The detection of gravitational waves by PTA should open an additional venue for tests of general relativity. Tests of general relativity include tests of the memory effect. This effect is a permanent change in the strain of spacetime, created during the merger of a binary. After a very short overview of the theoretical foundations of the memory effect, the prospects for discovering this effect in the various current and future observatories will be presented. The existing constraints from ground-based detectors and the prospects for future observatories including space-borne instruments such as LISA will be summarised. Emphasis is then placed on discussing the specificities of the detection of the memory effect with the PTA as well as the recent constraints at current PTAs and those expected with SKAO.

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## 1 Introduction: the memory effects

The displacement memory effect refers to a permanent change in the relative positions of free falling test masses caused by the passage of a gravitational wave. This effect is a prediction of General Relativity.

The linear memory effect was first discussed in Zel'dovich and Polnarev (1974), Braginsky and Grishchuk (1985) and Braginsky and Thorne (1987). The first calculations were done in linearized gravity. The linear memory effect arises from a burst of energy-momentum flux when two astrophysical compact objects pass close to each other, typically from asymmetric mass ejections e.g., in supernovae or binary mergers.

The nonlinear displacement memory effect was then first demonstrated in Christodoulou (1991) and further discussed in Thorne (1992).The nonlinear displacement memory effect arises from the fact that gravitational waves themselves carry energy and momentum, and this energy flux contributes as a source of further waves, leading to additional distortions in spacetime. This effect is indeed nonlinear because it comes from the nonlinear nature of the gravitational interaction as described in General Relativity and depends on the self-interaction of the gravitational waves. Any source of gravitational waves will generate nonlinear memory. The non linear displacement memory effect is also sometimes referred to as the Christodoulou's effect although it was found earlier in Blanchet and Damour (1988, 1992) in the post-Newtonian formalism.

The displacement memory effect is related to asymptotically flat space-time symmetries in General Relativity known as the symmetries of the infinite-dimensional Bondi-Van der Burg Metzner-Sachs (BMS) group, see Bondi et al. (1962); Sachs (1962); Wald (1984), see also Compère and Fiorucci (2018); Compère (2019). The BMS group is a semi-direct product of the usual Lorentz group and of an infinite dimensional group of angle dependent translations [1] called supertranslations. The BMS group is larger than the known and usual Poincaré group of symmetries. The difference of the relative positions of free falling test masses before and after the passage of a gravitational wave i.e. the permanent change in their relative positions, corresponds to a BMS supertranslation.

Remarkably enough, there exists a connection between the memory effect and the so-called soft theorem originating from quantum electrodynamics and generalized to gravity in Weinberg (1965), see also Strominger (2017). In particular Weinberg (1965) showed that the infrared divergences arising in the perturbative quantum theory of gravitation can be removed by methods used in quantum electrodynamics. The infrared identities that have been derived have then been demonstrated to be the Ward identities of BMS symmetries in Strominger and Zhiboedov (2016).

Subdominant effects such as the spin memory and center of mass memory effects have also been found to take place in addition to the displacement effects as discussed in Nichols (2017) and Nichols (2018). Generalizations of the notion of the memory effects not necessarily connected to a symmetry has been performed in Flanagan et al. (2019), Flanagan et al. (2020) and Grant and Nichols (2022). See also the discussions in Flanagan and Nichols (2015), Compère and Nichols (2021) and Elhashash and Nichols (2021) for a more detailed discussion on definitions and impact of angular

[1] Angular position on the celestial sphere in asymptotically flat spacetimes.

momentum in asymptotically flat spacetimes.

In parallel, further extensions of the BMS group have also been proposed in Barnich and Troessaert (2010a), Barnich and Troessaert (2010b), Campiglia and Laddha (2015), Campiglia and Peraza (2020), Pasterski et al. (2016) Siddhant et al. (2024). See also Compère (2019).

One should also mention a whole line of discussion concerning gravitational wave memory in the context of expanding Friedmann-Lemaître-Robertson-Walker (FLRW) spacetimes and cosmology [2] i.e. see for example Chu (2015), Chu (2017), Tolish and Wald (2016), Bieri et al. (2017) and Jokela et al. (2022). It has been in particular argued in Tolish and Wald (2016) that, for sources at the same luminosity distance, the memory effect in a spatially flat FLRW spacetime is enhanced over the Minkowski case by a factor of $(1 + z)$. Besides, it has also been showed in Jokela et al. (2022) that, in a curved spacetime background, gravitational radiation develops a tail part arriving after the main signal that travels along the past light cone of the observer. This tail part slowly accumulates after the light cone part has passed and grows to a sizeable magnitude over a cosmological timescale. This tail part of the memory effect will be a new component in the stochastic gravitational wave background which effect remains to be evaluated - see for example Zhao and Cao (2022) for an attempt in this direction.

Interestingly, the role of spacetime dimension parity (spacetime with odd or even dimensions) on BMS supertranslations and the memory effect has been discussed in Hollands and Wald (2004), Hollands et al. (2017) and Garfinkle et al. (2017) where it is shown that there is no gravitational wave memory effect in a spacetime with dimension greater than four.

Finally, memory effects have also been discussed in the context of many scenarii beyond General Relativity. A comprehensive review of these extensions goes beyond the scope of this document but one can for example refer the reader to Heisenberg et al. (2023) and Heisenberg et al. (2024) for a theoretical discussion. Thus the measurement and discovery of memory effects would not only provide a remarkable test of general relativity and possibly a hint at the existence of fundamental symmetries of nature such as asymptotic symmetries but also, fascinatingly, provide insight into some of the most intimate properties of spacetime. So far, as of 2025, no displacement memory effects have been discovered in either LVK or PTAs. In the following sections we briefly discuss the status of these searches as well as the prospects at current and future ground or spatial observatories before discussing the case of PTAs/SKA.

## 2 The memory effect signal in black hole binaries

In the case of a coalescence of a black hole binary the gravitational wave signal is known to go through three consecutive phases i.e. an inspiral phase where the gravitational wave signal oscillates about a zero value and whose amplitude gradually increases, then a merger phase where

[2] As well as a whole discussion on BMS-like symmetries in such a context as can be seen for example in Kehagias and Riotto (2016) and also Bonga and Prabhu (2020) where it is argued that while asymptotically flat spacetimes have a specific mathematical structure at null infinity in particular in terms of the BMS group, in contrast, FLRW spacetimes are not asymptotically flat because their stress-energy tensor does not decay sufficiently fast, and, in fact diverges, at null infinity.

the amplitude of the signal increases significantly to reach a maximum and finally a ringdown phase where the signal returns to a zero value. In this sequence, the nonlinear memory effect usually shows up as a growing, nonoscillatory contribution to the gravitational wave where the signal amplitude returns to a nonzero value in the ringdown phase.

A sketch from Favata describing this sequence versus time is shown in Fig. 1.

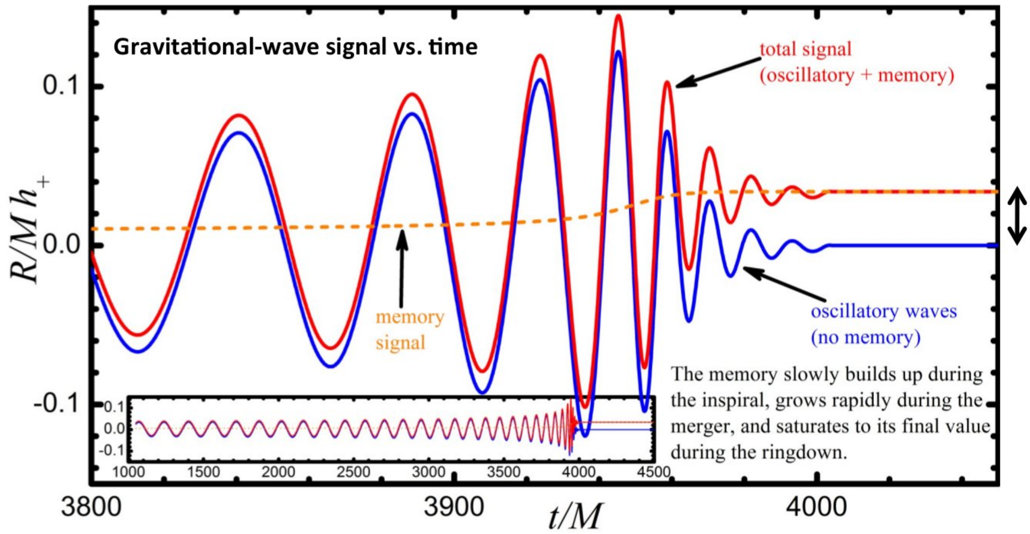


**Figure 1:** The inspiral, merger and ringdown sequence of a black hole binary with and without the memory effect. Figure taken from Favata. The vertical axis corresponds to the dimensionless, distance-independent waveform amplitude in the + polarization and the horizontal axis corresponds to the dimensionless time variable. $M$ stands for the total mass of the binary system and $R$ stands for the distance from the source to the observer.

The nonlinear memory effect can then be seen as a nonoscillatory modulation of waveform of the gravitational wave [3]. The nonlinear memory effect is also sometimes referred to as gravitational wave memory bursts in the literature. One could also in principle obsverve the memory effect even if the oscillatory part is not observed. The memory during the inspiral phase for quasicircular orbits has been first calculated in Wiseman and Will (1991) and Kennefick (1994). The nonlinear memory contributions were computed to 3PN order in Favata (2009b). The memory for the whole inspiral, merger, and ringdown sequence was then computed in Favata (2009c) and Favata (2011) (see also Favata (2009a)) where prospects for its detection at ground-based interferometers as well as in the space mission LISA were discussed. Dominant and subdominant memory effects as well BMS frame have started to be taken into account in numerical relativity and surrogate models as can be seen from Pollney and Reisswig (2011), Mitman et al. (2020), Mitman et al. (2021a) , Mitman et al. (2021b), Magaña Zertuche et al. (2022), Mitman et al. (2022) , Yoo et al. (2023) and Mitman (2024). Finally, calculations of gravitational-wave waveforms with memory from the oscillatory part of the waveform has been implemented in a package called GWmemory which includes surrogate waveforms i.e. see Talbot et al. (2018).

[3]Hence often referred to as an offset. This offset in the wave form caused by the nonlinear memory effect is maximum at the merger phase of the black hole binary coalescence.

## 3 Memory effects searches in ground based detectors

The prospects to detect gravitational wave memory from binary black hole (BBH) mergers at LIGO/Virgo/Kagra (LVK) received an additional boost from the onset of its data taking in the light of the first lessons learned from the first event GW150914 ( Abbott et al. (2016)), as discussed in Lasky et al. (2016). The discussion pointed to a possible detection by Advanced LIGO/Virgo with signal-to-noise ratio (SNR) of 3 (respectively 5) with an ensemble of $\sim 35$ (respectively 90) events with masses and distance similar to GW150914. One indeed would rely on the detection of an ensemble of merger events as the detection of the memory effect from an individual event is expected to be unlikely in view of the small fraction of the total strain it carries.

Yang and Martynov (2018) also discussed binary neutron stars (BNS) mergers as a possible source of gravitational memory in addition to BBH. In particular they studied BNS mergers with a simple memory waveform model which could be sensitive to the star's equation of state (EOS) thus opening the possibility to distinguish between different EOS with the memory effect. They concluded that memory effects in BBH as well as using these effects to shed some light on neutron stars EOS may be achieved with the third-generation ground-based detectors. Interestingly, Tiwari et al. (2021) showed that memory effects could even be used to distinguish BNS from BBH systems although yet inconclusive in the case of GW190425, see Abbott et al. (2020).

As data taking proceeds and the LVK gravitational wave transient catalog (GWTC) grows, more discussion on the search for the memory effect are taking place, although not coming directly from the LVK collaborations, and the above forecast sensibly evolves in some cases, see for example Boersma et al. (2020) (where they pointed out that different SNR can be obtained depending on the different approximation methods used to compute the GW memory waveforms) and see also Khera et al. (2021), Hübner et al. (2021), Zhao et al. (2021) and Grant and Nichols (2023), the latter also discussing the subdominant spin memory effect.

One of the most recent studies from Cheung et al. (2024), using the third LVK GWTC, points to a non observation of the memory effect with the current available LVK data. They further stressed that a catalogue of O(2000) binary black hole mergers would be needed to detect memory as already inferred in Hübner et al. (2021), which could occur during the the fifth observing run expected to start in 2027.

The case for memory effects (displacement, spin and others in the context of extended BMS and generalized BMS symmetries) of next generation ground base gravitational detectors such as Einstein Telescope, see Punturo et al. (2010), and Cosmic Explorer, see Reitze et al. (2019) has been discussed in Grant and Nichols (2023) and Goncharov et al. (2024) (the latter discussing prospects for LISA as well, anticipating on the next section). Goncharov et al. (2024) find that ET with $O(10^4)$ events with optimal SNR expected to represent of the order of 5% of the BBH merger events to be observed within a year would constrain the displacement memory strain amplitude to 2% and, to a lesser extent, the spin memory strain amplitude to 2.2 % at the $1\sigma$ level, where the displacement memory strain amplitude and the spin memory strain amplitude have been introduced as additional multiplicative factors to the predicted gravitational wave memory strain amplitude. Goncharov et al. (2024) also emphasize the importance of taking the displacement memory effect into account in order to improve parameter estimation as was already hinted by Gasparotto et al. (2023) in the case of LISA.

## 4 Memory effects searches in LISA and other space-based gravitational wave detectors

With a space-based gravitational wave detector such as LISA, operating at millihertz frequencies, the prospects for memory detection look even more promising.

The possibility to detect memory effects in LISA with supermassive black holes (SMBH) binary mergers was discussed in Islo et al. (2019) and Arun et al. (2022). Islo et al. (2019) considered SMBH binaries at $z < 3$ with masses in the range $(10^5 - 10^7)M_\odot$ and with mass ratios in the range $0.25 - 1$. They found that LISA could detect memory burst events with SNR greater than 5 at a rate of 0.3 - 2.8 times per year in the most optimistic galactical environmental effect model [4]. As pointed in Arun et al. (2022), this could be also seen as a lower limit as, for a 3-year LISA lifetime, most SMBH binaries with SNR greater than 5 in their inspiral phase lie beyond $z = 3$, see for example Sesana et al. (2007), Klein et al. (2016) and Seoane et al. (2023).

As mentioned above Goncharov et al. (2024) also explored the memory effect detection with LISA as well as its potential ability to give information on the symmetry groups of spacetime, be it BMS or extended BMS.

Morover, following up Gasparotto et al. (2023), more comprehensive and quantitative results have been obtained in Inchauspé et al. (2025) based on more realistic and detailed simulation of the LISA instrument as well as recent SMBH population models, for the latter see Barausse and Lapi (2021) and Barausse et al. (2020) (based on earlier work from Barausse (2012), Sesana et al. (2014) and Antonini et al. (2015)). From the analytical derivation involving spin-weighted mode decomposition of the GW amplitude, it can be shown that the memory is essentially showing up in the $(2, 0)$ mode since most of the primary GW energy flux is released in the $(2, \pm 2)$ mode, see for example appendix A of Inchauspé et al. (2025). Focusing on the GW memory contained in the $(2, 0)$ mode of the GW waveform and assuming four years of observation with LISA, SNR studies were performed with different scenario in terms of mass ratio, spin and sky localisation. Contour plots of constant SNR in the redshift versus total mass plane were derived for the so-called primary wave, the memory component and the SNR ratio between the two. The SNR ratio was in particular shown to be up a few percent in a conservative scenario with mass ratio of 2.5 and zero spin. The impact of spin and mass ratio has also been studied where it has been showed that the SNR memory strongly decreases with the mass ratio and that it increases for aligned spins. As shown in Inchauspé et al. (2025), LISA is expected to have a different relative sensitivity for the memory and the primary wave depending on the sky localization. Sky-localization maps of the primary wave and memory modes SNRs as well as their ratio has then also been provided showing the sky

[4] Islo et al. (2019) used a simple power-law to model possible galactical environmental effects which could influence the SMBH merger timing, and thus the time at which the memory effect is maximum, as can be seen for example in Begelman et al. (1980) for the infamous final parsec problem (see also however Alonso-Álvarez et al. (2024) for a possible solution) where the SMBH binaries could stall although recent PTA's evidence would rather point to the fact that SMBH binaries do actually merge.

localisation dependence of the two components and their ratio. Furthermore, using eight different astrophysical models of SMBH population corresponding to different astrophysical uncertainties affecting their evolution and leading to different event rates in LISA i.e. light or heavy initial seeds [5], different time delay model between the galaxy merger and the SMBH merger i.e. delayed or short delay [6] or taking supernovae effects into account or not, resulting into inhibiting or not the accretion of massive black holes, they were able to show that the detection prospects for LISA indeed vary with the population model, with heavy seeds models being more promising than light seeds models. Interpreting recent PTA results in terms of SMBH inspiral could point to a SMBH population model Barausse et al. (2023) which could be of an important value for the search for memory effects with LISA, see also Agazie et al. (2023c), Antoniadis et al. (2024b) and Agazie et al. (2023e).

It has furthermore been proposed in Hou et al. (2024) to use space interferometers and in particular the DECIGO mission (see Kawamura et al. (2019) and Kawamura et al. (2021)) to search for the memory effect from stellar-mass binary black holes (BBHs) which turn out to be abundant enough according to the knowledge acquired from the current ground-based interferometers.

## 5 Memory effects searches with PTA

The search for memory effects with the Square Kilometre Array (SKA) will obviously benefit from the experience acquired at current Pulsar Timing Arrays (PTAs) where the search strategies are still progressing (see or example Sun et al. (2024)) and are likely to continue to do so in the future.

Evidence for a stochastic gravitational background signal has been announced in 2023 in a serie of papers from the various Pulsar Timing Array (PTA) collaborations, see Agazie et al. (2023b), Antoniadis et al. (2023), Reardon et al. (2023a) and Xu et al. (2023).

This serie of papers have also been complemented by publications concerning searches of individual sources and/or continuous GW sources, see Agazie et al. (2023d) and Antoniadis et al. (2024a), as well as implications on SMBH, anisotropies, dark matter and the early Universe, see Antoniadis et al. (2024b), Agazie et al. (2023c), Agazie et al. (2023c) and Agazie et al. (2023e).

The detectability and sensitivity of PTAs to GW burst with memory have been discussed in many papers, see for example Pshirkov et al. (2010), van Haasteren and Levin (2010), Seto (2009), Finn and Lommen (2010), Pitkin (2012), Cordes and Jenet (2012), Wang et al. (2015), Madison et al. (2014) and Islo et al. (2019).

Specific searches for GW memory burst at PTA have been performed in Wang et al. (2015), Arzoumanian et al. (2015), Aggarwal et al. (2020) and more recently in Agazie et al. (2024), the latter with techniques described in more details in Sun et al. (2023).

To date, there is no significant detection of GW memory burst from PTAs and upper limits on the

[5] Namely light seeds of population III stars or heavy seeds from the direct collapse of protogalactic gas disks.

[6] Where the delayed time models aim at model processes at parsec distances whereas as the short delay models neglect these processes and aim at taking effects such as dynamical friction between halos. Delayed models are often considered more realistic.

GW memory strain amplitude have been derived in the presence of a common spatially uncorrelated red noise (CURN) as reported in Arzoumanian et al. (2020).

As discussed for example in Madison et al. (2014), mergers of SMBHB systems are most likely to be sources of GW memory burst in particular the one with mass ratios near one [7].

To briefly establish some orders of magnitude, following Pshirkov et al. (2010) itself building on Favata (2009a) and Reisswig et al. (2009), for a SMBH circular binary, for which the $\times$ polarization of the memory vanishes, and with equal masses $M_1 = M_2 = M = 10^8 M_\odot$ at a luminosity distance of $D_L = 1$ Gpc, the characteristic amplitude of a GW memory burst neglecting higher order contributions can be approximated as follow :

$$h_{\texttt{mem}} = 5 \times 10^{-16} \left(\frac{M}{10^8 M_\odot}\right) \left(\frac{1\texttt{Gpc}}{D_L}\right). \tag{1}$$

Thus, as mentioned in Madison et al. (2014), two $10^9 M_\odot$ BHs at a distance of 1 Gpc from earth is expected to produce memory with an amplitude $h_{mem} \approx 10^{-15}$ with optimal beaming.

This has to be compared with the typical amplitude associated with the inspiral phase prior to the SMBH merger which can be approximated by (see Sathyaprakash and Schutz (2009) as referenced in Pshirkov et al. (2010)) :

$$h_{\texttt{ins}} = 10^{-15} \left(\frac{M}{10^8 M_\odot}\right) \left(\frac{1\texttt{Gpc}}{D_L}\right). \tag{2}$$

which is roughly an order of magnitude higher than $h_{\texttt{mem}}$ from equation 1 for the same values of $M$ and $D_L$.

During the merger phase of the SMBHB coalescence, where the amplitude of the GW signal increases significantly to reach a maximum as mentioned above in section 2 and where several percent of the system's total rest-mass energy is radiated as GWs, the memory grows in a very sudden way compared to the typical timescale of the whole inspiral, merger and ringdown sequence. Namely, as discussed in Cordes and Jenet (2012) and Madison et al. (2014), the typical timescale for the memory to reach its maximum during the merger phase for a merger producing a $10^9 M_\odot$ BH is of the order of one day. This timescale is significantly smaller than the typical pulsar timing measurement cadences of several weeks.

Furthermore, as argued in Agazie et al. (2024), although the frequency of the GW emitted by these SMBHB systems during the final inspiral and merger phases is most likely to be outside the nanohertz frequency band, the corresponding accumulated memory may still give rise to a significant enough signal to be detected at PTAs.

[7] Other sources of GW bursts such as the one coming from cosmic strings have also been considered in the literature see for example Damour and Vilenkin (2000) which signals detection at PTAs may need to be disentangled from the one coming from memory bursts as discussed for example in Divakarla et al. (2020). This question would certainly request a longer discussion which goes beyond the scope of this section.

### 5.1 Memory signals at PTAs

It is known that GWs propagating between the pulsar and the observer lead to modulations in the observed frequency of the pulsar signal, see Grishchuk (1974), Estabrook and Wahlquist (1975), Sazhin (1978) and Detweiler (1979).

Recording the times of arrival (TOAs) of radio pulses from a collection of millisecond pulsars (MSPs), known to have very stable rotational periods and forming a pulsar timing array (PTA), is then expected to lead to the observation of timing residuals caused by the passage of these GWs.

Since the displacement memory effect affects the distance between the pulsar and the earth, one then expects that the TOA residuals of the pulses of the pulsar will in turn also be affected by a GW memory burst thus causing an additional change in the observed frequency of the pulsar.

As explained in Agazie et al. (2024), a GW memory burst passing over a single pulsar will induce a sudden increase or decrease of the observed frequency of the pulsar by a constant amount. In contrast a GW memory burst passing over the earth will induce a sudden increase or decrease of the observed frequency by a constant amount for all observed pulsars in the array.

These sudden changes by a constant amount will induce differences between the expected frequency and observed frequency of the pulsars which will make the timing residuals accumulate linearly during the time span of the pulsars observation.

For a GW memory burst passing over just one pulsar, only the timing residuals in the TOA pulses of that pulsar will be derived whereas for a GW memory burst passing over the earth the timing residuals of the TOA pulses of every single observed pulsar will accumulate.

The timing residuals induced by a GW memory burst $\delta h_{\mathtt{mem}}(t)$ can be expressed as the product of two terms, see for example Sun et al. (2023) and Agazie et al. (2024) :

$$\delta h_{\mathtt{mem}}(t) = B(\hat{\mathbf{k}}, \hat{\mathbf{p}}, \psi) \times h_{\mathtt{mem}}(t), \tag{3}$$

where $B(\hat{k}, \hat{p}, \psi)$ is a projection factor which represents the relative orientation of the source and the pulsar for a GW memory burst propagating in the direction $\hat{\mathbf{k}}$ and polarization angle $\psi$ passing through the line of sight of a pulsar at sky position $\mathbf{p}$ and where $h_{\mathtt{mem}}(t)$ is the time dependent amplitude of the GW memory burst.

This time dependent amplitude of the GW memory burst can be expressed as follow :

$$h_{\mathtt{mem}}(t) = h_0 \Big[ (t - t_0)\ \Theta(t - t_0)\ -\ (t - t_i)\ \Theta(t - t_i) \Big], \tag{4}$$

where $t_0$ is the time at which the GW memory burst passes over the earth, $t_i = t_0 + r_i[1 + \cos\theta_i]$, is the retarded time at which the same GW memory burst passed over the $i^{\mathtt{th}}$ pulsar where $r_i$ is the earth-$i^{\mathtt{th}}$ pulsar distance at an angle $\theta_i$ to the direction of the wave propagation. $\Theta$ is the Heaviside function and $h_0$ corresponds to the intrinsic strain of the memory signal.

The term $h_0(t-t_0)\ \Theta(t-t_0)$ in equation 4 is called the earth term whereas the term $h_0(t-t_i)\ \Theta(t-t_i)$ is called the pulsar term.

As the one day timescale of a typical GW memory burst at a SMBHB mergers is significantly smaller than the typical pulsar timing measurement cadences of several weeks, one expects that the detailed turn-on of the memory signal is not sufficiently resolved which, in turns, makes equation 4 sufficient to describe the memory signal as stressed in Arzoumanian et al. (2015).

Furthermore since the typical distances $r_i$ to each pulsar in the currently ongoing PTAs experiments are of the order of a kiloparsec (i.e. thousands of light years) and the typical observation time is tens of years, a GW memory burst will likely only be detected in the earth term or the pulsar term, but not both. This means that only one of the two terms of the r.h.s. of equation 4 will be non zero.

As stressed for example in Arzoumanian et al. (2015) and as the timing perturbation from a GW memory burst will show up simultaneously for all pulsars in the PTA if it occurs in the earth term one generally considers more advantageous to search for GW memory bursts in the earth term rather than searches in pulsar terms.

### 5.2 Recent results from PTAs

Recent searches for GW memory burst include the Bayesian based search by the NANOGrav PTA in their 12.5 yr data set Agazie et al. (2024) followed more recently by a search with their 15 yr data set Agazie et al. (2025) which, at the timing of writing, happens to be the most recent one. The methodology used for these searches have been detailed in Sun et al. (2023). The most recent Nanograv PTA results on the GW memory burst are very briefly summarized in the following.

Having reported the presence of a common uncorrelated red noise (CURN) process in Arzoumanian et al. (2020), i.e. a systematic common process not spatially correlated across pulsars which could come from the instrument or from astrophysical sources not related to a GW, the search included a CURN modeled by a simple power law with an amplitude and a fixed spectral index [8].

Several gaussian red and white noises were also included such as a gaussian red noise for long-timescale changes in the pulsar's rotational frequency and gaussian white noise parameterized by three widely used parameters known as EQUAD, EFAC and ECORR where EQUAD represents an additional white noise component added in quadrature to the timing residuals, EFAC scales the uncertainties of the timing residuals after including EQUAD and ECORR accounts for correlated noise within a single observation epoch but uncorrelated between different epochs.

The earth-term Bayesian search for GW memory burst performed by the NANOGrav PTA with 15 years of data Agazie et al. (2025) uses MCMC sampling and, by computing a Bayes factor, compares a noise-only model including the above discussed intrinsic pulsar red noise, white noise, and a common red-noise processes, with a signal model including the same noise processes with an additional GW memory signal. In a first analysis, excluding 270 days from both ends of the

[8] Two possible values were considered in Agazie et al. (2024). One is 13/3 as expected for a stochastic gravitational wave background generated by an ensemble of circular SMBHBs. The other one is 5.5 for the maximum a posteriori value found for the spectral index of their CURN in Arzoumanian et al. (2020). For the most recent result in Agazie et al. (2025), only the value of 13/3 was reported.

data set, NANOGrav PTA reported a Bayes factor of 3.1 with a standard deviation of 0.3 in favor of including a GW memory burst around $t_0 = 58000$. However NANOGrav concluded that this Bayes factor was not significant enough to claim evidence for a GW memory event arguing that the corresponding event occurs in the last three years of their dataset where they have low sensitivity. In a second analysis which uses the same priors on burst epoch as the one use in their previous analysis with their 12.5 years data set, they found a Bayes factor of 1.22 with a standard deviation of 0.04 disfavoring the GW memory burst.

In absence of GW memory burst detection, pulsar-term and earth-term upper limits on memory strain amplitude as a function of sky location on the one hand and as a function of burst epoch on the other hand have been derived.

## 6 Expected improvements towards the SKA

Before the advent of the SKA, and in spite of the yet current negative results, the search for GW memory burst at current PTAs is going to continue with more data accumulating and with improvements expected in many fronts ranging from noise processes understanding, GW memory burst model description to analysis techniques to mention just a few.

These improvements, together with the construction of a larger pulsars catalog with improved sky coverage, are expected to enhance the sensitivity of the SKA to GW detection in general and GW memory burst in particular with respect to current PTAs.

The sensitivity of PTA's and the SKA to GW memory burst detection not only depends on the number of pulsars in the array with the best possible isotropy but also on the number of measurements and the precision of the timing-residual measurements van Haasteren and Levin (2010).

The TOA uncertainties from current PTAs typically ranges from few tens of nanosecond to several microsecond. The root-mean square of the TOA residuals are currently at the level of few tens nanoseconds at best for many MSPs Agazie et al. (2023a); Antoniadis et al. (2024a); Reardon et al. (2023b); Chisabi et al. (2025).

Several sources of noise originating from the pulsar itself, the observing telescope and the propagation in the interstellar medium (ISM) and interplanetary medium (IPM) affect the precision of the TOA residuals.

The noise from the pulsar itself encompasses the so-called jitter noise which is coming from the intrinsic variability in the shape of individual pulses from a pulsar. This noise could appear as a limiting factor leading to a TOA residuals precision floor of the order of few tens of nanoseconds. However, with larger telescopes such as the one foreseen in the SKA and with the observation of a large number of pulsars with jitter noise, one expects to significantly improve our understanding of this source of noise and thus improve the TOA residuals precision as mentioned in Janssen et al. (2015).

The noise originating from the observing telescope encompasses the noise from the radiometer as well as polarization mis-calibration, radio frequence interferences (RFI) and clock errors. Radiometer noise appears to limit the performances of current PTAs Babak et al. (2024). As emphasized in Pennucci (2019), the radiometer noise $\sigma_{rad}$ is proportional to the system temperature $T_{sys}$ and inversely proportional to the telescope effective area $A_{eff}$ times the square root of the product of the integration length $t_{obs}$ and the bandwidth $\Delta f$:

$$\sigma_{rad} \propto \frac{T_{sys}}{A_{eff}\sqrt{t_{obs}\Delta f}}. \tag{5}$$

Therefore, by acting on one or all of these last three parameters it would be possible to significantly reduce the radiometer noise. The development of RFI mitigation algorithms (e.g. real-time RFI filtering system Buch et al. (2023) with real-time adaptive RFI masking technique Morello et al. (2021), deep learning approaches Xiao Zhang (2024) or excision techniques Maan et al. (2021) Buch et al. (2019), to cite but just a few, see also Baan (2019) together with particular data analysis techniques for the creation of TOA (see below) are also expected to improve the quality of pulsar timing measurements Wang et al. (2024). In the realm of the SKA instrumentation development efforts one can also mention further RFI mitigation strategies such as Gunaratne et al. (2019).

The noise from ISM origin includes the diffractive interstellar scintillation, the (time-dependent) dispersion measure (DM) mis-estimation, the frequency dependent DM, and scattering and the one from (IPM) includes the solar wind DM mis-estimation.

ISM-originated noise mitigation techniques include broadband observations[9] with ultra-wideband receivers aiming, among other things, at making one-shot measurements of DM, low-frequency DM monitoring using separate low-frequency telescopes (e.g. LOFAR, SKA1-Low), as well as high-frequency observations aiming at suppressing both dispersion and scattering delays or dynamic spectrum analysis (see for example Levin (2015)).

In particular, in the case of SKA, SKA1-Low for example, is expected to operate from 50 to 350 MHz, while SKA1-Mid is designed to operate from 0.35 up to 15 GHz in multiple bands.

Overall SKA is expected to improve timing precision by about an order of magnitude Babak et al. (2024); Janssen et al. (2015) (see also Lazio (2013)) by reaching uncertainties below $\sim$ 100 ns in the pulsar timing measurement.

Pulse jitter may set an ultimate floor for most of bright MSPs seen with the SKA and one limiting factor governing timing precision could then come from the duration of the observation. However better precision down to few tens of ns could be achievable on short timescales i.e. with 5 (10, 15) mins integration time, which takes 0.7 (1.8, 3.3) day Wang and Mohanty (2018).

Techniques based based on principal component analysis have been discussed in Oslowski et al. (2011) and Oslowski et al. (2013) (see Britton (2000) for an early proposal) to mitigate the effects of the pulse profile variability on TOA estimations. Based on observations of J0437-4715, Oslowski et al. (2013) concluded that an rms timing of about 30 ns in 1 hour integration could be achieved.

[9] Hardware upgrades can also be complemented with algorithms developments as, for example shown in Pennucci et al. (2014) where improvements in simultaneous TOA + DM precision measurement using 2D frequency-phase template fits from folded wideband pulsar data have been reported. See also Lee et al. (2014) for a statistical method to correct variations in the dispersion measure.

On the other hand, improvements of analysis techniques can come from many fronts.

For example, GW memory burst model description could benefit from some specific prescription as hinted for example in the particular case of hyperbolic BH encounters in Dandapat et al. (2024).

Concerning data analysis pipelines, several options for the creation of TOAs relying on various templates approaches including template matching methods, data derived templates and smoothed templates for current PTAs have been discussed in Wang et al. (2022, 2023, 2024) and could be useful for the SKA. See also Pennucci (2019) for frequency-dependent template techniques.

Other improvements could also come for example from the use of model averaging instead of model selection in pulsar timing, see van Haasteren (2024). Strategies GW memory burst detection could be further developped where one suggestion could for example be to rely on the use of computationally efficient frequentist detection, see Sun et al. (2024).

Many more proposals are likely to emerge for current PTAs in the future which the SKA will certainly benefit.

Current sensitivity forecasts for the SKA have been discussed in Babak et al. (2024) and Hazboun et al. (2019) (see also Graham et al. (2015)).

More specifically, using the formulae for the GW memory burst sensitivity estimates from van Haasteren and Levin (2010), assuming 10 years duration for the SKA with a fairly large number of pulsars i.e. 200 pulsars isotropically distributed in the sky, with 250 timing-residuals measurements and timing precision of 100 ns (resp. 50 ns, 30 ns and 10 ns), one could roughly achieve a sensitivity of $6.7 \times 10^{-17}$ (resp. $3.3 \times 10^{-17}$, $2.0 \times 10^{-17}$ and $6.7 \times 10^{-18}$) on the expected measurement error of the burst amplitude.

## 7 Summary

The nonlinear memory effect is a prediction of general relativity, whose experimental demonstration using gravitational waves detection represents not only an important fundamental physics goal for current observatories as well as future ones, whether on Earth or in space, but also a formidable challenge.

After having briefly reviewed the constraints known to date from LVK and PTA, and briefly discussed the prospects for a space-borne instrument like LISA, we summarized the known advances that SKA could benefit from, both in terms of instrumentation quality and analysis techniques. These advances, combined with a substantial increase in the number of observed pulsars, could reveal the nonlinear memory effect, particularly for sources that are a priori inaccessible to LISA and LVK.

Mastering TOA accuracy represents a major and critical challenge in this area. Current PTAs are developing strategies to improve their performance in this regard, however, SKA appears particularly well-positioned to address this challenge even more decisively.

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